Generation of Novel Chord Progressions via a Musically-Inspired Chaotic Mapping

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Double Pendulum Chaotic System

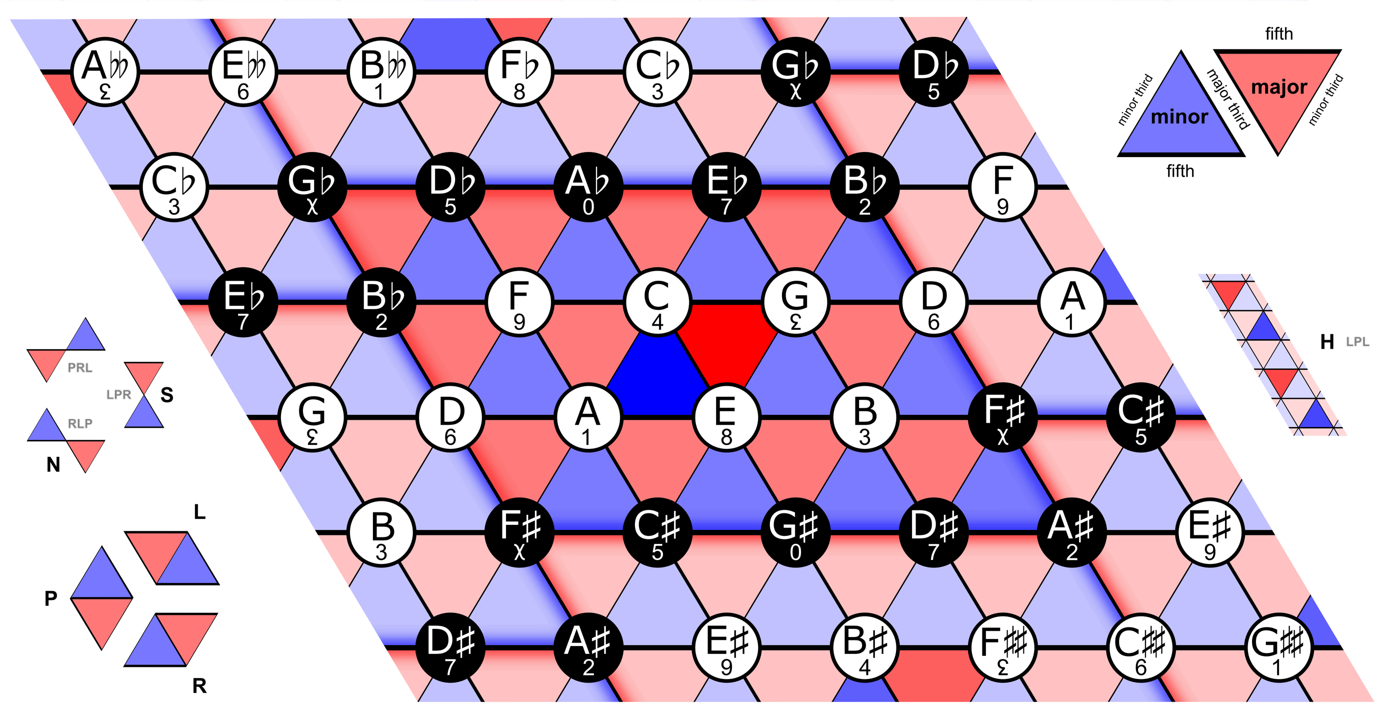
The double pendulum system has two periodic degrees of freedom, the angles of each mass θ_1 and θ_2, and angular velocities for each mass, yielding the system of differential equations, which is chaotic for most masses m_i, rod lengths ℓ_i, and initial conditions[3]:

$$
\begin{cases}
\dot{\theta}_1 = \omega_1 \\
\dot{\theta}_2 = \omega_2 \\
\dot{\omega}_1 = \dfrac{-g(2m_1 + m_2)\sin\theta_1 - m_2 g \sin(\theta_1 - 2\theta_2) - 2\sin(\theta_1 - \theta_2)\, m_2\left(\omega_2^2 \ell_2 + \omega_1^2 \ell_1 \cos(\theta_1 - \theta_2)\right)}{\ell_1 (2m_1 + m_2 - m_2 \cos(2\theta_1 - 2\theta_2))} \\
\dot{\omega}_2 = \dfrac{2\sin(\theta_1 - \theta_2)\left(\omega_1^2 \ell_1 (m_1 + m_2) + g\,(m_1 + m_2)\cos\theta_1 + \omega_2^2 \ell_2 m_2 \cos(\theta_1 - \theta_2)\right)}{\ell_2\,(2\,m_1 + m_2 - m_2 \cos(2\theta_1 - 2\theta_2))}
\end{cases}
$$

Highlights

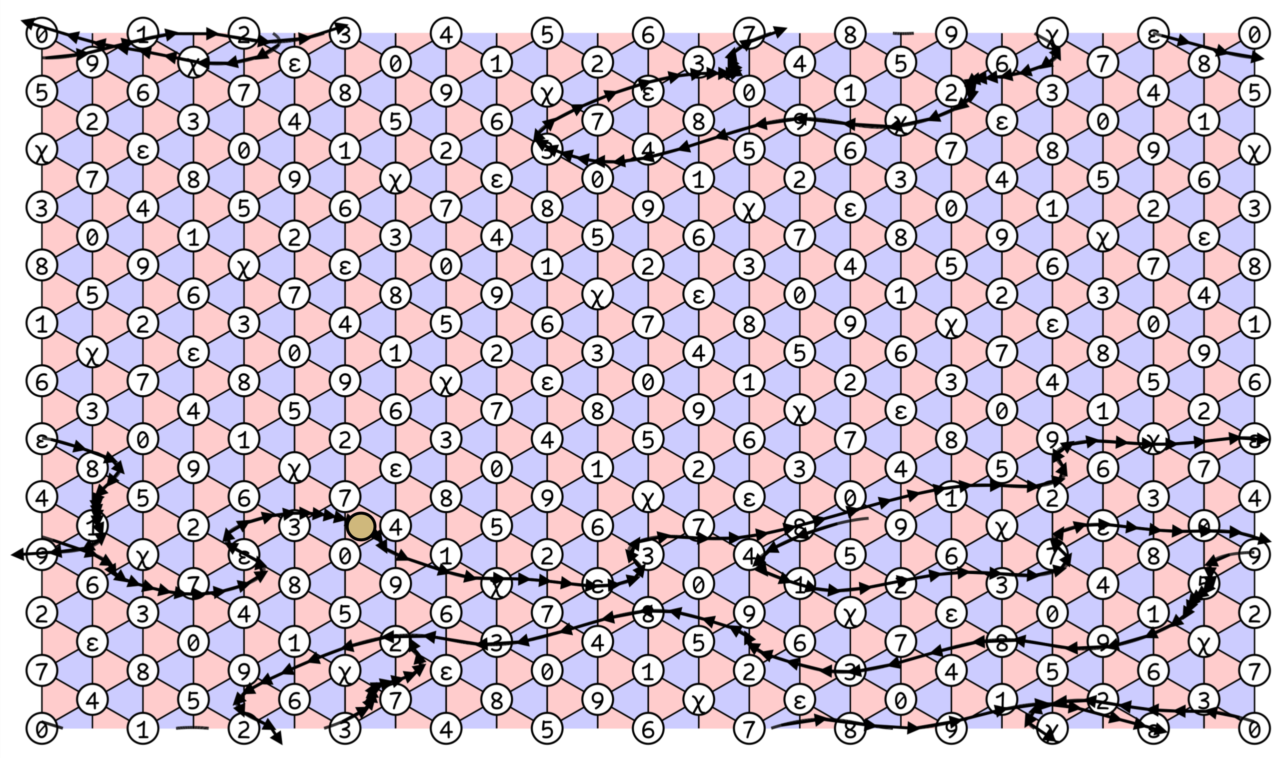
- We utilize a chaotic mapping onto a set of symbols to generate chord progressions
- Symbol arrangement is determined by the *Tonnetz*, where pitches are vertices of a doubly-periodic simplex mesh[4]
- Double-pendulum trajectory is mapped onto the *Tonnetz*, where the sequence of triangles visited determines chord progression
- Progressions are evaluated according to metrics[2] which correlate with positive listening experiences

The *Tonnetz*



Three periodic axes: modulo 3 (minor third), 4 (major third), and 7 (perfect fifth), image from [5]

Tonnetz-Inspired Chaotic Mapping



Pitch classes of a chaotic trajectory projected onto the Tonnetz. Chord at gold dot on trajectory is $\langle 0, 4, 7 \rangle$, that is, C-E-G or C-major.

Mapping Trajectories to the *Tonnetz*

Since our representation has repeated notes, we use the periodic DoFs modulo 12π and 16π rescaled to the unit interval, then interpolated onto the x and y axes of the Tonnetz:

$$
\begin{cases}
\theta_1 \mapsto \theta_1 \bmod 12\pi \big/ 12\pi \\
\theta_2 \mapsto \theta_2 \bmod 16\pi \big/ 16\pi
\end{cases}
$$

Mapping Points on the *Tonnetz* to Chords

Each point on the projected trajectory is mapped to a chord according to the vertices of its enclosing triangle on the *Tonnetz*.

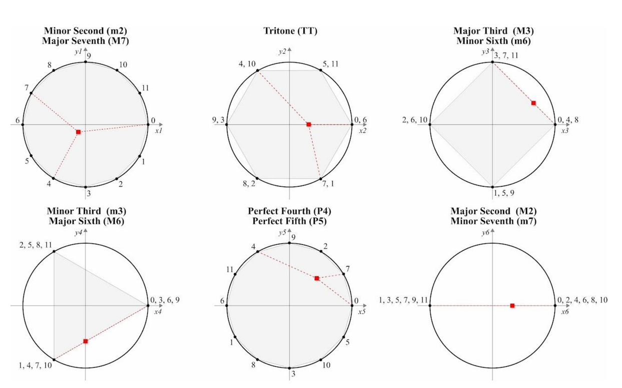
Analysis via Tonal Interval Space

Tonal Interval Vector[1] **(TIV)** of a chord is the weighted Fourier transform of its chroma vector $\boldsymbol{c} \in \mathbb{Z}_2^{12}$,

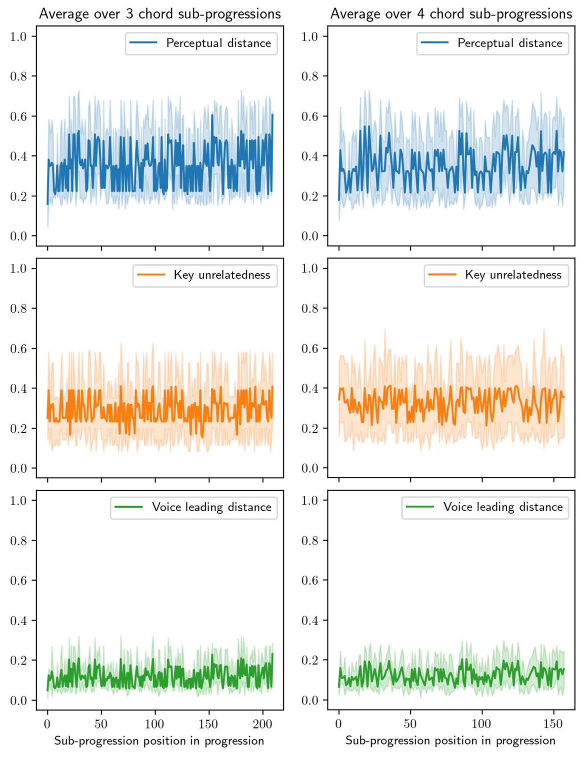
$$T_k = \frac{w_k}{\bar{c}} \sum_{n=0}^{11} c_n e^{-2\pi i \cdot nk / 12}, \qquad k \in \mathbb{Z}$$

where,

- w_k are weights based on music theory
- $c_n = 1$ if n is in the pc-set
- $\bar{c} = \sum_{n=0}^{11} c_n$ is the energy of the chroma vector



Visualization of correspondence between TIV entries and musical intervals, used from [1]



Perceptual Distance

Euclidian distance between TIVs of chords[2]:

$$d_p(\boldsymbol{T}_1, \boldsymbol{T}_2) = \|\boldsymbol{T}_1 - \boldsymbol{T}_2\|_2$$

Key Unrelatedness

Cosine distance between TIV of a chord and a key[2]:

$$d_k(\boldsymbol{T}, \boldsymbol{T}_{key}) = \cos^{-1} \frac{\Re(\boldsymbol{T} \cdot \boldsymbol{T}_{key})}{\|\boldsymbol{T}\| \|\boldsymbol{T}_{key}\|}$$

Voice Leading Distance

Measures how parsimonious two successive chords are, chords with fewer notes in common have higher distances[2].

Chord Progressions

Pitch Class Sets

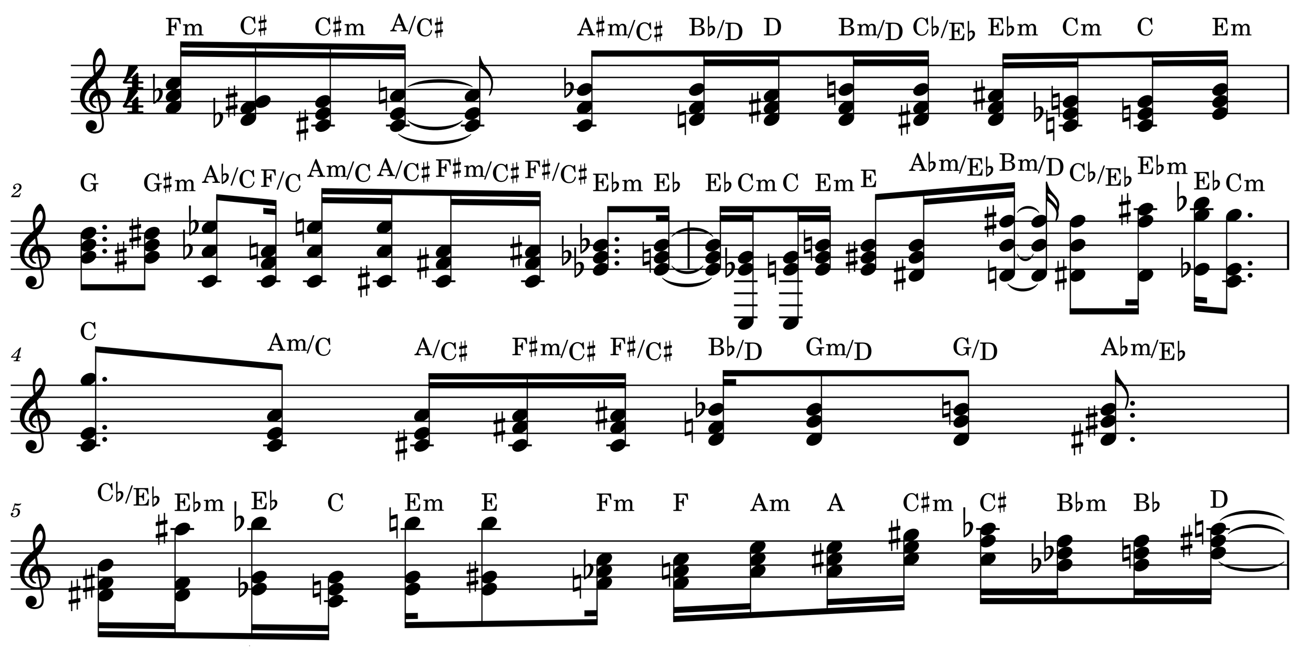
Vertices of enclosing triangle of *Tonnetz*, used for analysis

Octave

Three octaves, arranged to minimize voice leading distance, used for playback

Duration

Amount of time in triangle, rounded to nearest 16th note, used for playback



Listen Here!

Conclusions & Next Steps

We present a method for generating novel chord progressions without the use of preexisting works employing a musically-inspired chaotic mapping. While the variety of chords is limited to only major and minor triads in the current work, extensions to the *Tonnetz* permit for the addition of four-note chords, greatly improving chord variety[4]. Additional extensions include the generation of melodies with strong voice leading via of the geometric dual to the Tonnetz, a grid of hexagons wherein each represents a single note[4]. Our work also indicts the efficacy of "objective" metrics of musical quality, as our high performance under the tested metrics does not necessarily correspond to pleasant listening experiences.

[1] Bernardes, Gilberto, Diogo Cocharro, Marcelo Caetano, Carlos Guedes, and Matthew Davies. "A Multi-Level Tonal Interval Space for Modelling Pitch Relatedness and Musical Consonance." *Journal of New Music Research* 45 (May 27, 2016): 1-14. https://doi.org/10.1080/09298215.2016.1182192.
[2] Navarro-Cáceres, María, Marcelo Caetano, and Gilberto Bernardes. "Objective Evaluation of Tonal Fitness for Chord Progressions Using the Tonal Interval Space." In *Artificial Intelligence in Music, Sound, Art and Design*, edited by Juan Romero, Anikó Ekárt, Tiago Martins, and João Correia, 150–64. Lecture Notes in Computer Science. Cham: Springer International Publishing, 2020. https://doi.org/10.1007/978-3-030-43859-3_11.
[3] Shinbrot, Troy, Celso Grebogi, Jack Wisdom, and James A. Yorke. "Chaos in a Double Pendulum." *American Journal of Physics* 60, no. 6 (June 1, 1992): 491–99. https://doi.org/10.1119/1.16860.
[4] Tymoczko, Dmitri. "The Generalized Tonnetz." *Journal of Music Theory* 56, no. 1 (2012): 1-52.
[5] Image courtesy T. Piesk under CC0